State of the Art in the Realistic Modeling of Plant Venation Systems

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The modeling of plants is an active area of research in computer graphics and the geometrical modeling of veins is essential for obtaining plant images with a realistic appearance. The venation patterns determine the anisotropy of plant tissues, and affect the shadowing and masking of light incident on these tissues, hence the images. The current trend in the area of plant image generation is to aim for realism using biologically-based and predictable algorithms. Such algorithms do not depend on ad hoc parameters that have to tuned whenever a new image is being generated. A number of plant image generation algorithms are now available in the literature that have achieved an impressive level of sophistication in many aspects, generating realistic looking images. One notable exception is the geometrical modeling of veins. This remains as an open problem in computer graphics. In this paper we review the state of art of the realistic simulation of plant venation systems from a geometric modeling perspective and propose a specified set of requirements for evaluating possible solutions for this problem.

Keywords: plant; leaf; vein; biophysically-based modeling; predictive rendering.

1. Introduction

The venation system is an important feature of a plant leaf. A realistic portrayal of a plant must therefore include a realistic representation of the veins of its leaves.

Computer graphics is mostly concerned with creating realistic images of the world around us. Such images are either generated in an ad hoc manner or they have a physical and biological basis. Current research has a focus on the latter way of generating images since this allows the simulations to be used in a predictive manner [1]. Furthermore, this approach has a much broader applicability than picture making. Neither the ad hoc generation of images, nor the physically and biologically based direction of generating images have been completely successful in creating realistic images of veins, however.

In the computer graphics industry an artist or a technician is often employed to draw a branching vein structure as a texture which is then pasted onto a leaf model. This is an awkward and costly process. Because natural scenes are so frequently represented, it would be useful to devise an automatic and predictable technique for simulating leaf venation systems which can be embedded onto a leaf's geometric model, preferably based on fundamental physical and biological principles.

In this paper the problem of the automatic generation of a venation system is examined and the current state of the art is reviewed. Besides the direct implications in modeling, this problem has also implications in rendering since the presence of veins affects the way light interacts with plant tissues. The predictive modeling of veins may contribute to the accuracy improvement of plant reflectance models, which are essential not only for realistic plant image synthesis, but also in remote sensing simulations of radiation transfer to and from regions of vegetation [2]. Since plants represent a natural resource on which all human and animal life depends, such simulations are central in environmental studies (*e.g.*, projects oriented towards ecology, agriculture and forestry).

The remainder of this paper is organized as follows. The next section will provide a background on relevant biological concepts. Section 3 will discuss the requirements for an acceptable modeling technique for plant veins, and Section 4 will examine several potential solutions, analyzing these based on the stated criteria.

2. Background in Botany and Definition of Terms

The fields of biology and botany have a specific terminology to define objects and phenomena within their domains. This section therefore define terms and concepts from biology and botany relevant to this paper used in the sequel.

The main body of a leaf is called the blade, or the lamina [3] (Fig. 1). The blade begins from the petiole, which is the stalk of the leaf, and extends to its apex, or tip. A blade and its venation system does not exist in isolation. It is connected to a plant via a stalk known as the petiole. From the main stalk of the plant nutrients are transported via arrangements of plant vascular tissues known as xylem which conducts water and dissolved nutrients upwards from the root to the apex or the tip of a leaf. As the xylem progresses in the form of vascular bundles through the leaf it forms the venation system. Phloem is the name given to the plant vascular tissue which conducts sugar and other metabolic products downwards from the leaves, and it similarly form part of the venation system.

The veins lie in the mesophyll, a "diffusing and pigmented structure" [4] which provides the photosynthetic ground tissue of the leaf.

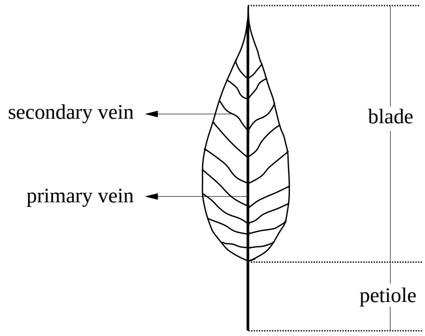

Fig. 1. Schematic representation of a plant leaf.

As well as providing a channel for nutrient conduction, venation systems provide the structural framework for the leaf. The venation system is also a very important component in the growth process of a plant. Furthermore, a leaf's venation pattern is useful in species recognition or classification, and is often considered as a plant's "fingerprint" [5,6,7,8,9,10,11,12,13].

2.1. *Venation Patterns*

Whenever there are objects with similar functions displaying a range of forms scientists turn to classification since the hope is that classification will lead to insights. This is also true with respect to venation patterns.

In fact there is a great variety of venation patterns across different species of plants and a classification in the form of a branching structure is often employed [5]. Relatively high in this structure it is possible to distinguish two main classes of plants, namely monocotyledons and dicotyledons. Monocots have one cotyledon, or seed leaf, whereas dicots have two. Monocot leaves have a parallel or striate venation system (Fig. 2), in which equally sized primary veins traverse the lamina lengthwise, coming together at the blade's apex. Smaller veins connect the larger parallel ones, and also run parallel to each other [14].

Light reflection is radially anisotropic with respect to reflection since experiments performed by Woolley [4] suggest that a leaf reflects light more diffusely when viewed from a direction perpendicular to the veins. This means that the simulation of light interaction with monocotyledons with their parallel vein structure is simplified since their anisotrophy is constant over the leaf whereas for dicotyledons it varies depending on the location on the leaf [15].

In dicots, venation is described as netted or reticulate (Fig. 2). Dicot veins follow a branching pattern in which the central vein, or midrib, begins from the petiole and traverses the leaf, branching into successively smaller, secondary and tertiary veins in a recursive fashion [14]. The secondary veins often extend to the edges of the leaf, and tertiary and higher-order veins create a complex network throughout the mesophyll [16], creating polygonal subdivisions of lamina called areoles. Within these areoles even smaller veinlets come to their ends in the mesophyll, within approximately 0.25 millimeters of each other [16]. An observation of the branching angle between subveins usually ranges between 30 to 60 degrees in dicots. The dicot's branching venation patterns are those most typically associated with plant leaves, and they are more difficult to generate than the parallel venation of monocots. As such, the pattern generation portion of this paper focuses on dicot veins.

Fig. 2. Photograph showing leaves with different venation patterns, namely striate (left) and reticulate (right).

2.2. *Thickness and Pigmentation*

The presence of veins in a plant leaf alters its surface shape and its pigment concentration, which in turn affects its interaction with light. The veins do not reside on the surface of the leaf (this is often done in computer graphics), but inside the mesophyll layer, under the cuticle and epidermis layers. However, the veins do protrude from the leaf surface, adding to the thickness of the leaf at the vein sites. This fact has numerous ramifications in computer graphics [17] and solid modeling. First, it affects the required physical representation. A silhouette view of a plant leaf's edge would need to show the physical protrusions in order to achieve realism. Secondly, veins affect the masking and shadowing of light on a leaf since the physical presence of veins creates obstacles for the light interacting with the foliar tissues (Fig. 3). From a geometrical point of view the leaf and the veins have both 3D properties as discussed above and 2D properties since the vein patterns may be projected onto a 2D surface with no vein projections crossing.

The different level of pigmentation found in veins has further consequences in the area of light interaction with the leaf. McClendon and Fukshansky [18] estimated that the light flux through veins was 10 times that of the flux through the neighbouring dense mesophyll of the leaf, because of lower concentration of pigment in

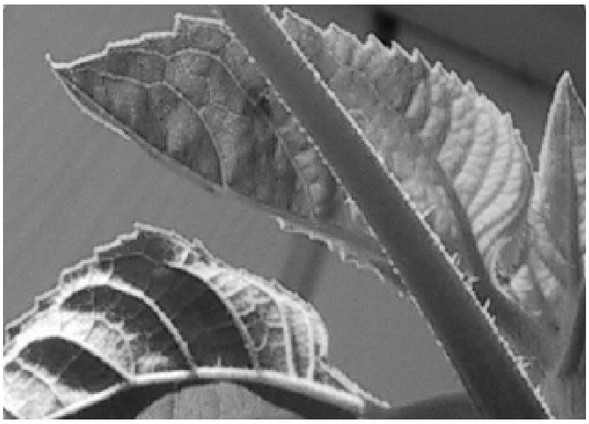

Fig. 3. Photograph showing self-shadowing of incident light by the veins.

veins. Furthermore, as mentioned earlier, a leaf's anisotropy depends on its venation pattern.

3. Requirements for a Biologically-Based Solution

Investigating plant leaves from the botanical perspective also provides important insights for the biologically-based modeling of plant leaf venation systems. In the previous section we summarized some results from botany which describe various factors affecting the appearance of these venation systems. The goal in realistic modeling of leaf veins is, therefore, to find a solution that will capture as many of these observable traits as possible. From the above examination we can glean several requirements which must be fulfilled by an automatic and predictable representation of leaf veins which claims to be biologically-based.

(i) The ultimate goal of a simulation is to automate the procedure for creating the pattern of the veins, including their 3D shape. Employing the talents of an accomplished artist or a skilled technician for drawing the veins clearly does not qualify as such a procedure. The procedure must provide a way for designing the dicot veins' branching pattern automatically, taking into account the above-described angles and inter-vein spacings in the mesophyll. It must also include the gradation of the thickness of the veins as they branch and approach their terminal points. As the venation pattern differs significantly between species, the procedure should also allow for parameters which can modify the pattern. These parameters are linked to the overall type of species, their habitat and geographical location. The solution must give an appropriate volume to the structure, taking into account the alteration in size between primary and higher-order veins, and their trajectories and smooth/bulged branching. A representation using straight, constant-sized cylinders might seem to be possible, but it is too simplistic since it does not allow for slowly varying thickness, nor for changes in direction. Joining cylinders of different diameters poses a problem as well. The solution must also provide for the variations between the very visible main branching patterns in different species of dicots.

(ii) The solution must allow the simulated veins to protrude from the surface of a leaf since a close view of the silhouette of a leaf should show their protrusion.

(iii) The method used must allow the simulated veins to interact correctly with light in the manner described in the previous section. This means that the modeled veins should include variations in pigment and thickness to enable the simulation of masking and shadowing effects, and anisotropy.

(iv) Independently of the method used to create the simulated veins, the solution must allow for changing level of detail. For a closely viewed leaf, the veins must be simulated in minute detail in order to capture all the physical aspects discussed above. For scenes where there are a number of leaves (for example, a house plant in an interior scene), the protrusion of the veins is not a noticeable effect and thus an equivalent texture map (possibly generated from the geometry itself before it is removed) may replace the geometry. In scenes with many plants or threes (for example a view of a forest) the detailed simulation of the veins will have no effect on the image and it is sufficient to consider the consider the effect of the veins on the overall illumination of a leaf.

(v) If the simulation of the venation system consists of pasting the veins onto a previously created leaf surface then it must provide a way to allow for movement or growth of the leaf, with the vein remaining "attached" to the surface and with both moving together realistically. This is an interesting problem, and an elegant solution is presented as the next criterion.

(vi) The most common techniques used for the representation of veins (Section 4) approach the problem of creating venation systems by "pasting" the veins onto a preexisting leaf model, so that the veins are a surface feature which depend on the leaf surface's placement and shape. In nature this dependency is reversed in the sense that the leaf veins are embedded inside the mesophyll layer, and the leaf tissue surrounds the veins so that nutrients may be delivered to the cells. This may seem to be a subtle difference with respect to computer graphics since the role of nutrient supplier is not important to vein representation, yet it does become important when the final leaf model is folded or otherwise manipulated. In current approaches, the veins would have to be made to follow the leaf, whereas a more biologically-based approach would use the veins themselves as a sort of skeleton which controls leaf movement. Bloomenthal [19] points this out by stating that model design "makes more sense inside out". This "inside out" design is more intuitive and realistic, and as such could handle the challenge of leaf movement more gracefully. Yet this final criterion applies to very sophisticated solutions where great effort is being made to ensure biological realism, and effective-looking solutions may exist which ignore this requirement. The main implications are in leaf movement and in leaf growth, which are topics beyond the scope of this paper.

4. Computer Graphics Approaches

A number of existing computer graphics techniques can be applied to this problem. Seven of these techniques are now examined and tested against the criteria discussed in the previous section. It should be noted that the most basic representations, considered first, fail on most of the criteria outlined above. More robust solutions are given in the following approaches.

4.1. *Texture Mapping and Bump Mapping*

Texture mapping is the simplest technique available for dealing with the representation of venation systems, and it is therefore one of the most commonly used methods in industrial computer graphics applications involving plants. To implement this method an artist will manually create the leaf venation system as a texture and map this texture onto the leaf model. The texture for the venation maps representing the primary and secondary venation systems can also be obtained through the scanning of real specimens. For instance, Fig. 4 shows plant leaves whose veins were simulated using the scanned texture maps presented in Fig. 5.

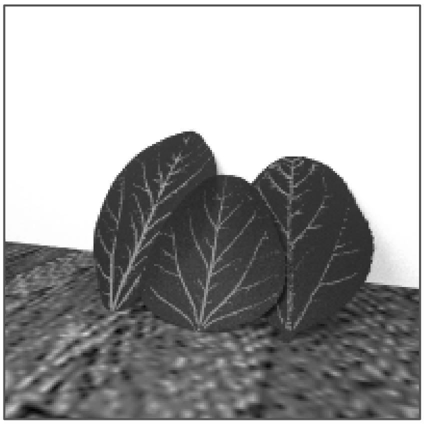

Fig. 4. Computer generated image of plant leaves using texture mapping.

Clearly, texture mapping alone is not an acceptable solution to the problem. The creation of the texture is manual and ad-hoc and not predictable, and even if the texture could be automatically generated, then a simple pasting of a texture onto a model does not provide a precise enough representation of the venation system and the leaf in that it does not allow for realistic light interaction with the object during rendering. Also, texture mapping does not solve the problem set out in the second

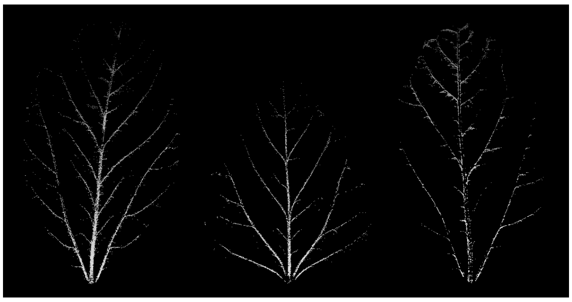

Fig. 5. Texture maps obtained by scanning real plant leaves.

criterion above, of portraying the protrusion of the veins when the silhouette of the leaf is viewed closely.

Bump mapping, or the manipulation of surface normals according to the surface features the texture is trying to convey, may be introduced in order to cause light to interact with the model roughly as it would if real modulations existed on the surface. Although this is a distinct advantage and can be accomplished relatively easily, it also fails to provide an adequate solution. As in texture mapping, the side view of the leaf blade does not show veins protruding from its surface. Also, some lighting effects such as shadowing and masking will still not be easily available, since a surface geometry is not physically present (Fig. 3).

4.2. *Displacement Mapping*

Displacement mapping can be regarded as an extension of texture and bump mapping, however, unlike the above methods it is capable of producing results which are realistic with respect to many of the above-stated criteria. Displacement mapping resembles bump mapping in that a map of alterations is created for an object model, which will be applied as the object is rendered. However, the displacement map does not only contain perturbations of surface normals, but also adjustments for the height of the surface at each point in the map. The method consists of constructing a scalar height field $h(x, y)$ whose values are added to points on the model's surface, $s(x, y)$, during rendering [20]. Thus, this method allows for the addition of geometry to a model when the model rendered. In traditional displacement mapping, these height values can only be added to the model's surface in the direction of the surface normal $N(x, y)$. This means that the surface resulting from rendering is $S'(x, y) = S(x, y) + h(x, y)N(x, y)$ [*ibid.*]. Pedersen's approach of using 3D flow fields to form the direction of displacement [21] provides a way to bypass the restriction on displacement direction if needed. However, his refinement is not needed for the purposes of veins on the leaf blade. For this problem, adjustment in the normal direction is adequate, and greatly reduces the storage needed for this algorithm. In the terms used in solid modeling the displacement mapping is a generalized offset procedure.

Displacement mapping is a relatively simple way of adding modulation to an underlying smooth surface. Since it creates a real geometry, it will cause correct interaction with light, including shadowing and masking effects. The geometry produced will also be visible in a silhouette view of the blade. The method also suggests a simple procedure for changing the level of detail in a scene: if only a textured representation of veins is needed the height map could be interpreted as a texture in some predefined manner. Also, although this method is among the simplest available, it does satisfy the fifth criterion. Because the offset surface is added to the underlying surface values, the veins will be correctly attached to the leaf surface, even if it is not flat. They will remain attached to the leaf when it moves or deforms. This solution may be the optimal choice for some applications since it is not expensive computationally and still satisfies many of our criteria.

Several drawbacks exist for displacement mapping with respect to the criteria mentioned. First, this method gives no direction as to how the map itself will be generated. Setting these heights based on data carefully obtained from a real leaf does not satisfy the requirement that the procedure be automated. The sections below will examine some available methods for automatically generating such maps. Secondly, like its predecessors, texture and bump mapping, displacement mapping deals with a venation system as a surface feature instead of as an essential component of the leaf's structure.

Another difficulty with this method is in determining the sampling density needed to represent the height map. Large sections of leaf with very small (invisible) veins should not be sampled as heavily as sections at the edges of veins. To deal with this problem, Wang proposes a method called "feature-based" displacement mapping for localizing sampling density based on discovered features in the displacement map [20]. His method uses the computation of a feature metric and an orientation for each point in the low-density sample. Discarding the sample points which are not meaningful, the procedure then adjusts the locations of remaining sample points to focus on the regions where features are determined to lie.

4.3. *L-Systems*

Lindermeyer systems, or L-systems, use a context-sensitive grammar to automatic generate plant geometry and to portray branching patterns in trees [22]. They satisfy the first criterion in that they can automatically design predictable branching patterns. A realistic pattern can be achieved and the grammar can be written to account for differences arising between species.

However, some geometry must be created to implement this pattern and it is not immediately clear how to do this realistically. A branching structure of line segments or straight cylinders will be too geometric and not give the impression of a natural structure. Algin developed a "generalized cylinder" which can be swept across a space curve and can vary in size of shape of its cross-section [19]. Because of these qualities, the generalized cylinder is ideal for representing natural forms. It may be used to bring geometry to the L-system, or else the L-system may be created using simple line or curve segments and serve as input to some other method described below.

L-systems are an important step in the direction of a solution for this problem, but several issues must be addressed if we intend that they should satisfy all of our criteria. For instance, we must develop a method of attaching this L-system to the surface of the leaf blade, and ensuring that it lies correctly inside the bounds of the leaf margin. A possible approach is to use the blade model as input to the process of L-system development to provide constraints for the grammar.

A related approach is taken in the paper [23] where a semi-probabilistic model for the growth of a ramified leaf structure is proposed.

4.4. *Cellular Texture Basis Functions*

Another procedure for generating a texture is the use of a cellular texture basis function as suggested by Burge [24]. Feature line segments can be used to link cells in a parent-child hierarchy, and various techniques may be applied to create leaf-specific features such as tapering towards the apex. Burge successfully modeled a texture for plant veins using this approach.

Another benefit in Burge's approach is his suggestion for extending this texture to a 3D model. He discusses placing feature geometry in each of the cells, which would allow a geometry to be rendered for the generated pattern [*ibid.*]. Is is unclear how this would be done or how precise the result would be, but this is a promising direction for future research. Such an approach would satisfy the criterion for shifting level of detail, since a textured leaf could be reinterpreted to obtain the geometry representation when the viewer moves closer to the leaf.

4.5. *Implicit Surfaces*

A promising method for vein representation is found in the idea of implicit surfaces. As proposed by Bloomenthal [19], the design of a model using implicit surfaces is achieved from the inside out, basing an outlying surface on an internal skeletal structure. Bloomenthal argues that this method is particularly suitable for the representation of natural forms, in which structure and movement depend on some form of skeleton.

Implicit surfaces are defined by functions which specify a volume surrounding a skeleton. Points in space which are roots to these functions are considered to lie on the object's surface. Thus, implicit surfaces are useful in skeletal design since the distance from skeleton to the outermost tissue is volumetric [*ibid.*].

The approach can be useful in two ways. First, it can automatically construct a geometry for veins based on a preexisting pattern made up of simple line or curve segments. This pattern could be generated by use of L-systems or some other method. Even if an L-system uses line segments to create the skeleton, Bloomenthal shows that the implicit surface based on such a skeleton will still look natural.

Concavity in the skeleton will result in tangent discontinuity in the surface, which appears in the form of an unnatural crease [*ibid.*]. This can be remedied by blending primitive volumes instead of taking the union of the surfaces. The blends produce bulging instead of creasing where two pieces intersect [19], which produces a more natural effect for plant veins.

The second potential use for implicit surfaces is as a way of basing the leaf blade structure and movement on its venation system. To achieve this, the function which defines the skeleton would be the automatically-generated branching pattern, and the surrounding implicit surface would be the leaf blade which envelops the veins, with raised bumps directly over vein lines. This effect may be achieved by the blending of two implicit surfaces based on the same skeleton, one for the veins themselves and one for the blade.

Bloomenthal used this approach, attempting to model the leaf itself as an implicit surface on the vein skeleton, even recreating the leaf margin based on vein ending points. His method had considerable success, although the margins were not perfect, and his approach had the advantage of being automatically generated by the vein branching pattern. He dealt only with primary veins and created a texture for the higher order veins. This texture was created by "exploding" the blade's triangle mesh into triangles, whose edges he warped slightly to give a more natural impression [19]. The spaces in between these exploded triangles became the texture for small veins on the leaf. There is much ingenuity in his method; it is efficient and does represent a branching structure bounded within the leaf. Yet his texture-generation method does not allow for sufficient control over the branching pattern achieved, and certainly does not allow us to differentiate the pattern between species of plants.

In general, however, Bloomenthal's approach has great promise. The use of implicit surfaces for the leaf blade gracefully satisfies our last criterion for the blade's dependence on the venation system. Movement of the entire leaf is made simple by automatically assigning weights for a certain vein's influence on the neighbouring surface. His method also allows for altering levels of detail. In combination with the use of generalized cylinders and blending, we can achieve realistic-looking geometries for the veins themselves.

4.6. *Particle Systems*

Structured particle systems have been used to devise models for natural forms such as trees or clouds. Instead of drawing a family of particles constantly in motion, the entire trajectory of each particle over its lifetime is drawn [25]. Thus structured particles can be used to create a static image.

The main benefit of structured particle systems for our problem is that this single method can be responsible for both pattern and geometry generation. The system can be structured so as to force the particles to follow a constrained branching pattern, and to remain on the level of the leaf surface at every point in its path. The particles can be given a spherical shape so as to have a semi-sphere protruding from the leaf at each point on a vein. Their diameter can be made to decrease with time, or with each bend in its path. The particles can die when their diameter falls below a certain threshold, or be explicitly killed off when they come within a few millimeters of the leaf margin.

Witkin and Herbert [26] propose a general method for combining particle systems with implicit surfaces. In their approach, particles and a surface are constrained together, while both the particles and the surface may move. The particle-surface constraints are based on classical mechanics and their method involves solving for derivatives of either the particles or the surface, given the velocity of the other's movement. This interesting method fulfills the last two criteria in constraining veins to the blade, even when the leaf moves or changes shape. With this method, leaf motion can also be initiated by the particles representing the venation system – the veins may be used to move the blade. However, this system is complex and designed to handle models for which real-time evaluation of changes must be done. If movement of the leaf is not a major priority, this solution may be more computationally expensive than necessary and the result may look less accurate than desired because of the use of particles.

4.7. *Cellular Texture Generation*

Another method allowing predictable generation of textures along with physical surface features was suggested by Fleischer [27]. Cellular texture generation was developed as a way of producing textures to model surface details. It combines a biologically-motivated cellular development simulator with a constraint to keep the surface features located on a preexisting surface. Fleischer intended this method to be used in cases where texture maps are insufficient, but where by-hand model manipulation is inconvenient. His method is based on "cells", which resemble particles in that they have surface characteristics such as local curvature and colour, but whose behavior depends heavily upon its neighbouring elements and the "extracellular environment". These cells interact as they mature in their environment, making used of reaction-diffusion properties and adhesion of cells to each other [27]. All these factors and cell constraints combine to form a first order differential equation term which will modify the cell state. An ordinary differential equation is created from the combination of cell terms and environment variables.

Benefits of this approach are that it is biologically motivated and can take into account environmental factors or properties of the underlying surface. It also guarantees that the cellular texture will remain attached to the underlying surface. The cells have constraints which can be used to make them repel each other at certain points, or stick together at certain points, potentially based on which region of the surface they lie on. Fleischer's method produced convincing representations of furry of thorny surfaces. It may not be straightforward to adapt the cellular development simulator to accommodate patterns as structured as venation branching, but the idea is worth examining. The major drawback of this method is that it treats veins as a surface feature instead of as a basis of leaf structure.

4.8. *Further Speculative Approaches*

Although we excluded leaf growth in Section 3, it might be that in order to achieve high realism in the representation of veins, a process based on the growth of leaves with the venation system included would provide the ultimate solution. This research would start with the examination of the growth of leaves for a specific species. Based on this study at least two stages of development would have to be identified. The first stage would be that of establishing the structural patterns of the main veins and the second stage would mimic the growth of the smaller veins, the veinlets, as the leaf develops. In this development, computational geometry techniques might be useful in that the veinlets develop in such a manner that they fill the space between the main veins in the most efficient manner, *i.e.*, keeping the maximum possible distances between them and the veins. This could then be combined with an implicit surface technique (Section 4.5) to create the required volumetric shape of the leaf.

Recently the study of venation patterns have been compared to the study of cracks [28,29,30]. It might be possible to use the results of this research to create patterns for computer graphics purposes as well.

A second approach is to use results and techniques from transportation research [31,32]. It was noted that the veins were vascular pathways for transporting nutrients to cells and waste products away from the cells. From this point of view the creation of a venation system can be abstracted to ask the question of what is the most efficient distribution network to accomplish the above tasks given the constraints of diameter of veins, length of veins, maximum pressure, entry point for the service etc.

A possible procedure would consist of

(i) define the outline (boundary of the leaf),
(ii) considering the average cell size define cell centers and generate the Voronoi diagram for the centers as approximations to the cells,
(iii) define major veins,
(iv) create an optimal branching network servicing all the cells with nutrients (work in progress).
(v) use implicit techniques to create the actual veins.

In this approach the growth of leaves would have to be considered since this would add constraints to the solution that would not be present in the static case. This is the case for all evolutionary transportation networks.

5. Conclusions

In this paper we described several approaches to the problem of a plant venation system representation, and we proposed a specified set of requirements to ensure that the solution is biologically-based and realistic. Although some of these approaches have never been used as a means of creating venation systems, they present different aspects that could be explored to fulfill the requirements listed. Investigating these techniques within the context of leaf venation systems will create an opportunity to assess their drawbacks and advantages more closely, and will contribute to further the state of the art in the realistic and predictable modeling of plant venation systems.

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References

1. D.P. Greenberg, J. Arvo, E. Lafortune, K.E. Torrance, J.A. Ferwerda, B. Walter, B. Trumbore, P. Shirley, S. Pattanaik, and S. Foo. “A framework for realistic image synthesis”, in *SIGGRAPH Proceedings, Annual Conference Series*, 477-494 (1997).
2. S. Jacquemoud and S.L. Ustin. “Leaf optical properties: A state of the art”, in “*8th International Symposium of Physical Measurements & Signatures in Remote Sensing*”, 223-332 (2001).
3. J.C. Semple. *An introduction to fungi, algae and plants: their morphology, classification and phylogeny* (Pearson Custom Publishing, Needham Heights, MA, 2nd edition, 1999).
4. J.T. Woolley. “Reflectance and transmittance of light by leaves”. *Plant Physiology*, **47**, 656-662 (1971).
5. E. Klucking. *Leaf Venation Pattern Vol. 1: Annonaceae* (Gebruder Borntraeger, Berlin, 1986).
6. E. Klucking. *Leaf Venation Patterns Vol. 2, Lauraceae* (Gebruder Borntraeger, Berlin, 1987).
7. E. Klucking. *Leaf Venation Patterns Vol. 3, Myrtaceae* (Gebruder Borntraeger, Berlin, 1988).
8. E. Klucking. *Leaf Venation Patterns Vol. 4, Melastomataceae* (Gebruder Borntraeger, Berlin, 1990).
9. E. Klucking. *Leaf Venation Patterns Vol. 5, Combretaceae* (Gebruder Borntraeger, Berlin, 1991).
10. E. Klucking. *Leaf Venation Patterns Vol. 6, Flacourtiaceae* (Gebruder Borntraeger, Berlin, 1992).
11. E. Klucking. *Leaf Venation Patterns Vol. 7, The classi cation of Leaf Venation Pat- terns* (Gebruder Borntraeger, Berlin, 1995).
12. E. Klucking. *Leaf Venation Patterns Vol. 8, Euphorbiaceae Part I* (Gebruder Borntraeger, Berlin, 1997).
13. E. Klucking. *Leaf Venation Patterns Vol. 9, Euphorbiaceae Part II* (Gebruder Borntraeger, Berlin, 2003).

14. P. Raven, R. Evert, and S. Eichhorn. *Biology of Plants* (W.H. Freeman, New York, sixth edition, 1999).
15. G.V.G. Baranoski and J.G. Rokne. "Efficiently simulating scattering of light by leaves", *The Visual Computer*, **17**(8), 491-505 (2001).
16. B.G. Bowes. *A Colour Atlas of Plant Structure* (Manson Publishing, 1997).
17. G.V.G. Baranoski and J.G. Rokne. *Light Interaction with Plants*, (ACM SIGGRAPH Course 26, IEEE Press, 2002).
18. J.H. McClendon and L. Fukshansky. "On the interpretation of absorption spectra of leaves - II. the non-absorbed ray of the sieve effect and the mean optical pathlength in the remainder of the leaf", *Photochemistry and Photobiology*, **51**(2), 211-216 (1990).
19. J. Bloomenthal. Skeletal Design of Natural Forms. PhD thesis, Department of Computer Science, University of Calgary, January 1995.
20. C.X Wang C.X., J. Maillot, E.L. Fiume, V. Ng-Thow-Hing, A. Woo, and S. Bakshi. "Feature-based displacement mapping", in *Rendering Techniques 2000*, ed. B. Peroche and H. Rushmeier (Springer-Verlag, Wien, 2000), 257-268.
21. H. Pedersen. "Displacement mapping using flow fields", in *SIGGRAPH Proceedings, Annual Conference Series*, 279-286 (1994).
22. P. Prusinkiewicz, M. Hammel, and E. Mjolsness. *L-systems: From the Theory to Visual Models of Plants* (CSIRO Publishing, 1996).
23. T. Pöschel and H. Malchow. "A simple model for the growth of ramificated leaf structures", *Chaos, Solitons and Fractals*, **4**, 1883-1888 (1994).
24. T.E. Burge. "A branching cellular texture basis function" in *SIGGRAPH 2000 Conference Abstracts and Applications*, ed. T. Appolloni (ACM SIGGRAPH, New York, 2000), p. 181.
25. W.T. Reeves. "Particle systems - a technique for modeling a class of fuzzy objects", *Computer Graphics*, **17**(3), 359-376 (1983).
26. A. Witkin and P. Heckbert. "Using particles to sample and control implicit surfaces" in *SIGGRAPH Proceedings, Annual Conference Series*, 260-278 (1994).
27. K. Fleischer, D. Laidlaw, L. B. Currin, and A. Barr. "Cellular texture generation", in *SIGGRAPH Proceedings, Annual Conference Series*, 239-248 (1995).
28. S. Bohn, B. Andreotti, S. Douady, J. Munzinger, and Y. Couder. "Constitutive property of the local organization of leaf venation networks", *Physical Review E*, **65**(061914), 1-12 (2002).
29. Y. Couder, L. Pauchard, C. Allain M. Adda-Bedia, and S. Douady. "*Sur la similitude des morphologies des nervations des feuilles et de celles des fractures*", in *Rencontre du non-linéaire* (Paris Onze Editions, 2000).
30. Y. Couder, L. Pauchard, C. Allain, M. Adda-bedia, and S. Douady. "The leaf venation as formed in a tensorial field", *The European Physical Journal B*, **28**, 135-138 (2002).
31. A. Roth, V. Mosbrugger, and A. Wunderlin. "Computer simulations as a tool for understanding the evolution of water transport systems in land plants", *Review of Palaeobotany and Palynology*, **102**, 79-99 (1999).
32. A. Roth-Nebelsick, D. Uhl, V. Mosbrugger, and Hans Kerp. "Evolution and function of leaf venation architecture: a review", *Annals of Botany*, **87**, 553-556 (2001).

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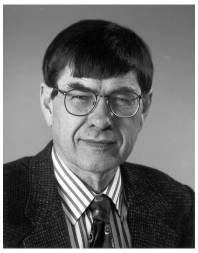

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