A Study on Sand Ripple Visualization

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Abstract

This report presents a comprehensive study on the visualization of sand ripple phenomena, focusing on aeolian ripples rather than subaqueous ones. We begin with an overview of the phenomena and categorization of sand ripples, driven by fluid dynamics and granular physics. Pioneering approaches, from procedural texture generation to physically-based and real-time simulations, are examined. Our discussion presents a comparison of existing algorithms, highlighting their strengths and limitations. The report concludes with perspectives on future research directions, exploring the potential of deep learning and physically inspired methods.

I. INTRODUCTION

Sand ripples are intricate patterns spotted in beaches and deserts, created and reshaped by the whims of wind and water. These natural sculptures, while often admired for their simple beauty, hold an abundance of scientific information. In this report, the phenomena is described while its visualization methods are presented and discussed.

Sand ripple formation has long been a subject of interest in the scientific community, offering insights into sedimentary processes and environmental conditions. Traditionally, researchers have relied on physical models and basic photographic techniques to study these patterns, but advances in technology and computational modeling have opened new frontiers. This study situates itself at the cutting edge of these developments, exploring innovative visualization techniques to capture and analyze sand ripples.

The structure of this report is outlined as follows. Sec. II explains what is the sand ripple phenomena and what kind of sand ripples exist. Sec. III surveys the recent methods used to visualize the sand ripple phenomena. Sec. IV discusses the methods and offers research perspectives.

II. SAND RIPPLE PHENOMENA

A. General Description

Sand ripples are intricate patterns formed on sand surfaces, primarily due to the repetitive action of water or wind over loose sand. Typically seen on beaches, deserts and other sandy terrains, they manifest as parallel ridges and troughs on the sand surface. Fig. 1 demonstrates the mesmerizing appearance of these sand ripples in a desert landscape. Most of the research and insights on sand ripple phenomena referenced in this section are derived from the seminal work of Bagnold (1974) on the physics of blown sand and desert dunes [1]. Sand ripples can be observed in a variety of settings, from coastal shores to expansive deserts and even in underwater regions with shifting sand beds.

Fig. 1: Photos depicting examples of sand ripples found in beaches (left) and dune fields (right).

The mechanism behind sand ripple formation involves the transportation of sand grains due to the fluid motion (either water or wind). As these grains are moved, they collide and aggregate, forming small piles. Over time, these piles grow and organize into periodic patterns known as ripples. The size, spacing and orientation of these ripples are determined by factors like grain size, fluid velocity and fluid consistency [1].

Grains typically range from 0.05mm to 2mm, with those on the lower end forming more delicate ripples, while larger grains result in broader, more robust formations. When the grain size is extremely small, the ripples can be subtle and harder to distinguish due to the sand's cohesive nature. Conversely, very large grains might not form well-defined ripples since they are less likely to be mobilized by fluid action.

B. Subaqueous Ripples

Bagnold and Taylor [2], pioneers in the study of sediment transport, particularly emphasized the understanding of ripple formation in subaqueous environments. Their work revolves around the two primary mechanisms for ripple formation under water: the "rolling-grain" and "vortex" ripples.

Rolling-grain ripples are formed primarily due to the motion of sand grains rolling along the bed under the influence of a slow flowing fluid. The continuous motion of water causes sand particles to gather and form ripples as seen in Fig. 2. In this category, the fluid's shear stress is just strong enough to move the grains, but not to lift them significantly from the bed. The grains simply roll over each other, aggregating in certain regions and forming ripples. These ripples are generally more regular and have smoother troughs and crests. They are often shorter in wavelength compared to vortex ripples because they are formed under weaker flow conditions, leading to smaller-scale, grain-by-grain sediment movement, as opposed to the larger, more turbulent sediment transport processes that create vortex ripples.

As the fluid flow becomes stronger or when the ripple size reaches a certain threshold, the flow dynamics begin to change. Over the lee side of the ripple (the side sheltered from the incoming flow), vortices begin to form as seen in Fig. 3. These vortices are essentially small whirlpools or swirling motions in the fluid. As they form and move, they play a role in lifting and moving the sediment grains. At this point, the primary mechanism of grain movement transitions from rolling to vortex-induced movement. These swirling motions exert shearing forces on the sediment bed, periodically lifting and redepositing sand grains.

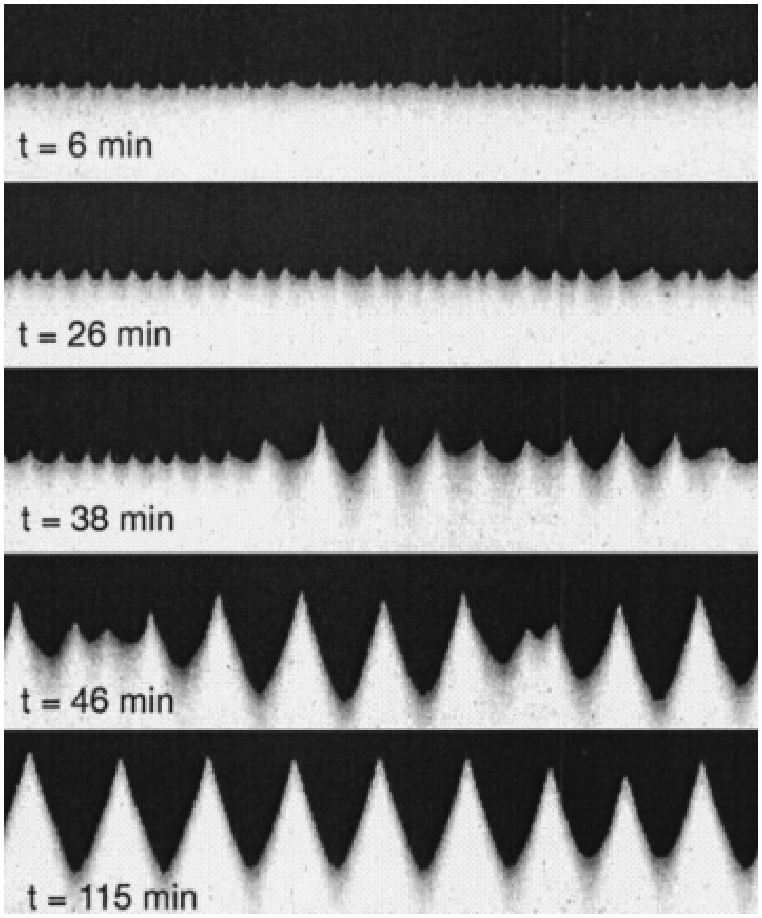

Fig. 2: Evolution of the rolling-grain ripple shape over time t under an oscillatory flow, where the vertical scale is multiplied by 9 with respect to the horizontal scale. Adapted from [3].

Over many wave cycles, vortex ripples are formed through this iterative process of suspension and deposition of grains. The crest of the ripple becomes crescent-shaped or lunate, pointing downstream. This is a distinguishing feature of vortex ripples. The presence of these crescentic features indicates a dominant flow direction, as opposed to the more symmetrical rolling-grain ripples which might not clearly indicate flow direction.

C. Aeolian Ripples

Bagnold [1], who was also was a pioneer in aeolian ripples, described the ripple formation in wind-blown sandy environments, later refined by Anderson and Haff [5]. Recent models [6] [7] delineate three primary mechanisms: saltation, reptation and avalanching.

Saltation is the primary mechanism for sand transport in aeolian environments, accounting for 95% of the bulk transport of sand in dune fields [8]. Sand grains “jump” or “hop” due to the wind’s force (Fig. 4). Similar to aqueous ripple formation, aeolian ripples require the wind’s drag and lift force to be strong enough to combat the grain weight. As grains are lifted by the wind, they follow parabolic trajectories and impact the ground, causing movement of other grains. This continuous hopping and landing of grains, combined with the wind direction, leads to the formation of ripples as well as reptation.

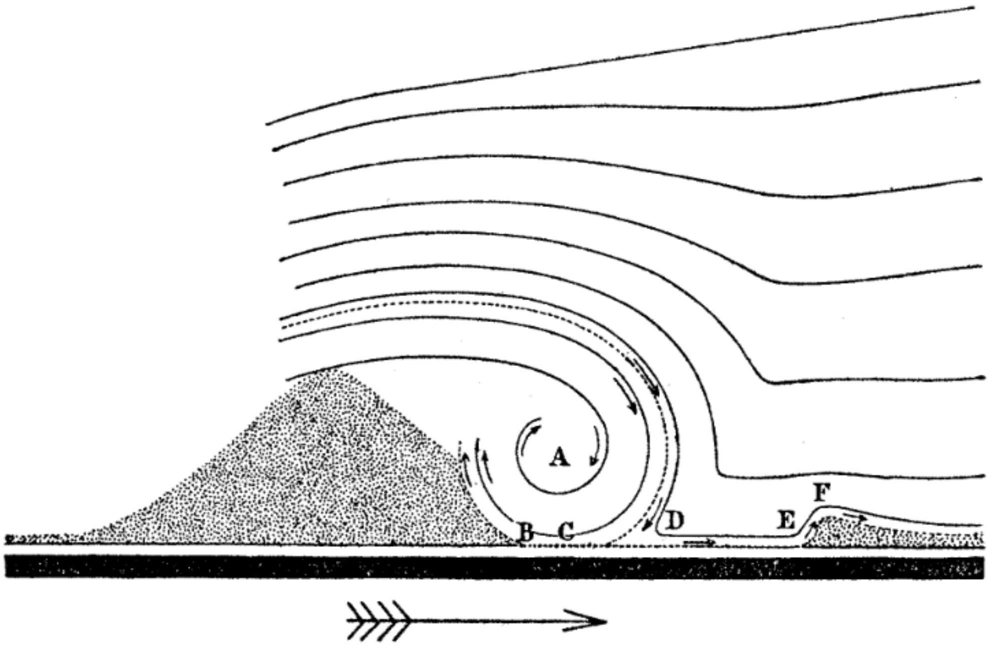

Fig. 3: As water continues to pour over the ridge, the vortex enlarges and the vortex ripple forms. The vortex carries the water flows A, B and C that are closer to the sand, whereas other impacted water flows D, E and F remain in their original direction. Adapted from [4].

Reptation, also known as creep, refers to the movement of sand grains due to the impact of other saltating grains as shown on Fig. 4. When a saltating grain (a grain that is in a “jumping” or “hopping” motion due to wind) lands on the sand bed, it ejects several smaller grains from the surface. These ejected grains, which move in short, low trajectories, cause a ripple pattern on the sand surface. Reptation ripples are smaller in scale, and their formation is dictated by the consistent bombardment of the surface by the saltating grains.

Suspension refers to the process of the the fluid capturing sand particles and moving them indefinitely into the fluid flow until the velocity drops. It is the other process shown in Fig. 4. Although suspension is a primary mechanism of sand transport, and its motion has been studied [9], it often receives less attention in visualizations due to its computational complexity and lesser visual impact.

Avalanching occurs when the local slope of the sand is greater than a given threshold defined by the repose angle as shown in Fig. 5. This movement results in a miniature “avalanche” as sand particles cascade down the slope, and contributes to the gradual reshaping of the ripple. Over time, this process leads to a characteristic angle of repose when equilibrium is reached. It is important to note that, unlike saltation, both reptation and avalanching do not necessarily have to move in the direction of the wind.

To summarize the phenomena, in both subaqueous and aeolian categorizations, the underlying theme is the delicate interplay between fluid dynamics (whether it is water or air) and granular physics. The ripple patterns we observe are direct manifestations of these interactions and provide a window into understanding both present and past environmental conditions.

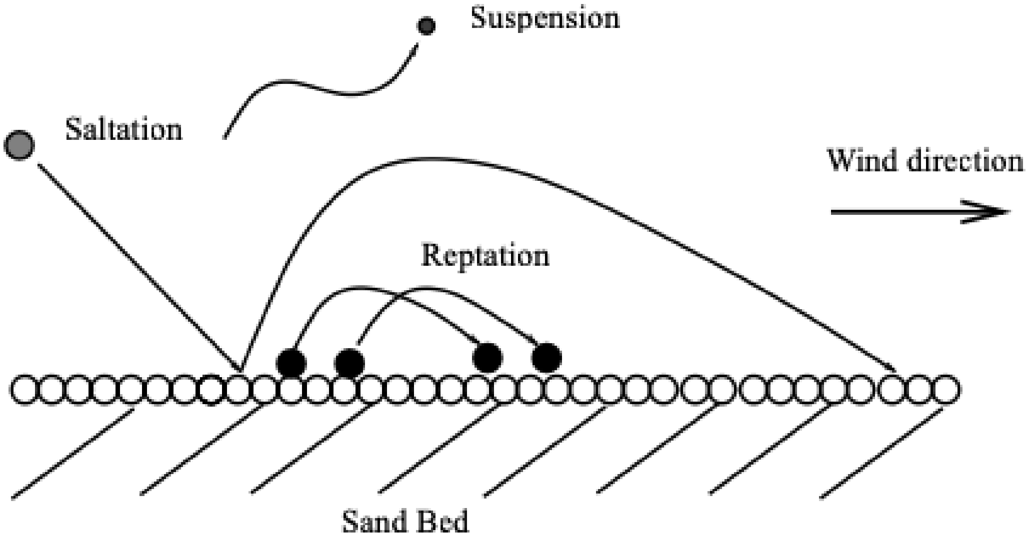

Fig. 4: Representation of several processes in aeolian ripples. The saltating grain jumps due to the wind's force and causes reptation, whereas the suspended grain is removed from the sand bed indefinitely. Adapted from [6].

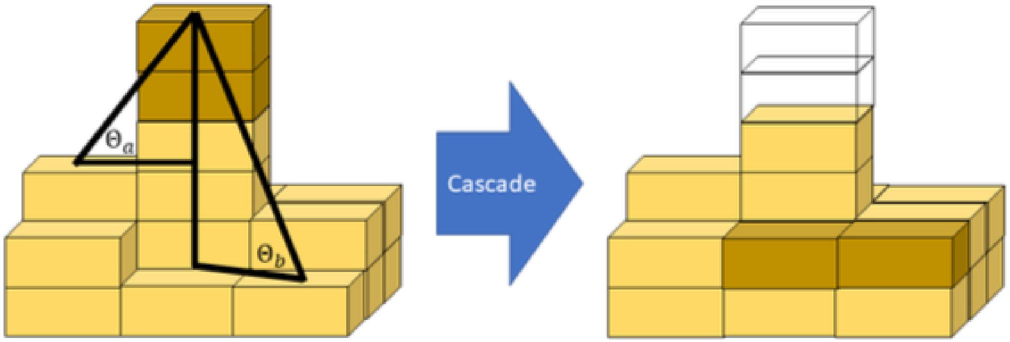

Fig. 5: A simplified representation of avalanching when an angle Θ_b exceeds the repose angle Θ_a, where the lowest neighbors receive cascading grains of sand. Adapted from [10].

III. A SURVEY OF THE STATE-OF-THE-ART

This part of the report delves into the historical development of various methods used to synthetically represent sand ripples. We begin in subsection III-A with an overview of the early techniques in simulating sand ripples procedurally. Next, in subsection III-B, we investigate the application of approximative models inspired by physical principles. In subsection III-C, our focus shifts to an in-depth analysis of methods grounded in physical based rendering. Subsection III-D is dedicated to advanced models capable of real-time depiction of extensive desert landscapes.

A. Procedurally Generated Textures

Sand ripples, as a subject of scientific study, have been of interest for centuries [4]. One of the earliest systematic studies of sand ripple formation was conducted by Bagnold [1] in the first half of the twentieth century. His applications of fluid dynamics to the movement of sand were groundbreaking, and his work still provides a basis for the study of sediment transport and pattern formation in granular materials today. Since the simulation of natural phenomena like sand ripples is an interdisciplinary challenge that involves computer science and physics, the history of simulating such phenomena in computer graphics is relatively recent, coinciding with the development of computational power and rendering techniques.

In the pursuit of simulating natural textures, Witkin and Kass [11] introduced a significant advancement with their work on reaction-diffusion (RD) systems in computer graphics in 1991, inspired by a biological pattern formation model proposed by Turing [12] in the 1950s. Even though sand is not typically regarded as a crucial component in biological organisms, it can be viewed as a morphogen in RD systems to procedurally create sand ripple visualizations.

The RD model incorporates diffusion, dissipation and reaction processes to simulate the distribution and interaction of morphogens. Diffusion is responsible for the transport of morphogens from areas of higher concentration to lower concentration, while dissipation accounts for the breakdown of these substances, leading to an exponential decay in their concentration. Reaction refers to the rate of morphogen production. To generate surface textures, the model proposed by Witkin and Kass assumes that diffusion occurs through a two-dimensional medium, although it can be applied to any arbitrary number of dimensions. The RD equation governing the morphogen concentration C is then given by:

$$\dot{C} = a^2 \nabla^2 C - bC + R, \tag{1}$$

where $\dot{C}$ is the time derivative of C, $\nabla^2 C$ is the Laplacian of C defined by:

$$\nabla^2 C = \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}, \tag{2}$$

a is the rate constant for diffusion, b is the rate constant for dissipation and R is the reaction function governing C. We can then compute the concentration of the morphogen at the next timestep with a simple integration formula, known as Euler's method [13]:

$$C_{t+\Delta t} = \Delta t(M * C_t + R_t), \tag{3}$$

where t is the previously known time, Δt is the timestep and M is the 3×3 mask matrix obtained by manipulating the diffusion and dissipation components of Eq. 1 and expressed as:

$$M = \frac{1}{h^2} \begin{bmatrix} 0 & a^2 & 0 \\ a^2 & -4 - h^2 b & a^2 \\ 0 & a^2 & 0 \end{bmatrix}, \tag{4}$$

where h is the distance between adjacent samples.

Witkin and Kass [11] extended the traditional RD model by incorporating anisotropic[1] and spatially non-uniform diffusion to produce more complex textures such as sand ripples, as the original could only support isotropic diffusion. The new model introduces the Hessian matrix [14] defined by:

$$H_{ij} = \frac{\partial^2 C}{\partial r_i \partial r_j}, \tag{5}$$

where the vector $r(x, y)$ is used to account for the rate of diffusion being different along different axes. This allows the simulation to create more complex patterns that are directionally dependent, akin to the natural formation of sand ripples under the influence of water or wind. By integrating the Hessian matrix, the diffusion component in Eq. 1 becomes $C_{diffusion} = \sum_i \sum_j A_{ij} H_{ij}$, with A representing the diffusion matrix given by:

$$A = \begin{bmatrix} a_1^2 \cos^2\theta + a_2^2 \sin^2\theta & (a_2^2 - a_1^2)\cos\theta\sin\theta \\ (a_2^2 - a_1^2)\cos\theta\sin\theta & a_2^2 \cos^2\theta + a_1^2 \sin^2\theta \end{bmatrix}, \tag{6}$$

where a_1 is the diffusion rate in the principal direction [cosθ, sinθ] and a_2 is the diffusion rate in the principal direction [-sinθ, cosθ]. Then, for anisotropic diffusion, the 3 × 3 mask matrix M in Eq. 3 becomes:

$$M = \frac{1}{2h^2} \begin{bmatrix} -a_{12} & 2a_{22} & a_{12} \\ 2a_{11} & -4(a_{11} + a_{22}) - 2h^2 b & 2a_{11} \\ a_{12} & 2a_{22} & -a_{12} \end{bmatrix}, \tag{7}$$

where a_{11}, a_{12}, a_{22} are the three distinct elements of the symmetrical diffusion matrix A in Eq. 6.

These equations can be discretized and solved numerically to simulate the evolution of morphogen concentrations over time, which can be visualized to represent the formation of patterns like sand ripples using texture mapping [15]. To facilitate the use of RD systems in graphics, Witkin and Kass [11] also developed efficient algorithms suitable for computer simulations. The result of rendering a sand texture button is shown on Fig. 6. Although rendering natural textures like sand ripples was the main goal of Witkin and Kass' contributions [11], their work laid the foundation for subsequent research in

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[1] Anisotropic diffusion refers to the process where the rate of diffusion varies depending on the direction. Unlike isotropic diffusion, where substances spread uniformly in all directions, anisotropic diffusion allows for different diffusion rates along different axes [11].

texture synthesis and influenced the various applications in computer graphics where natural patterns and complex textures were required.

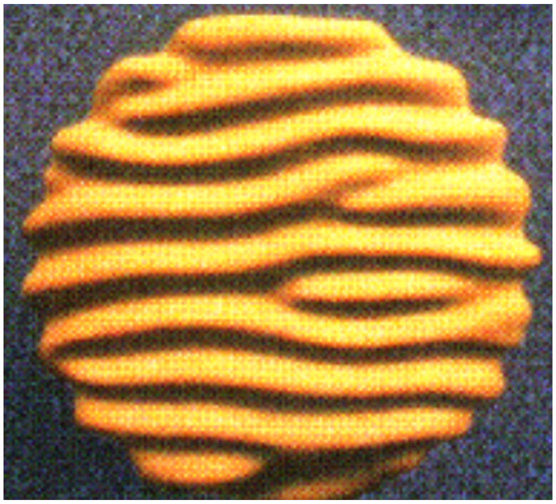

Fig. 6: A sand texture button rendered based on Witkin and Kass' RD system. Adapted from [11].

B. *Physically Inspired Appromixation Models*

Following the early advancements in RD texture synthesis, the field saw an evolution towards the modeling of larger scale natural phenomena such as deserts. This progression is shown by Onoue and Nishita's [16] seminal work in 2000, introducing a novel approach to the modeling and rendering of desert scenes, which until then, had not been a significant focus in computer graphics.

Onoue and Nishita were one of the first to apply Bagnold's [1] principles of saltation and reptation in the context of computer graphics, using them to inform the modeling of sand ripple dynamics. Onoue and Nishita's sand ripple model [16] is represented as a height field[2]. They define the saltation dynamics as:

$$h_{n'}(x, y) = h_n(x, y) - q, \tag{8}$$

$$h_{n'}(x + L(h_n(x, y)), y) = h_n(x + L(h_n(x, y)), y) + q, \tag{9}$$

where $h(x, y)$ is the height of the sand surface at each cell (x, y), q is the transferred height of grains, n is the timestep and n' is the intermediate timestep between n and $n + 1$. Note that this model assumes that the wind direction will always be in the positive x direction since the flight length of a saltating

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[2] A height field is a representation of a 3D surface where the height of the surface at each point is determined by a value stored in a 2D array [16].

grain $L(h_n(x, y))$ is approximated as:

$$L(h_n(x, y)) = L_0 + bh_n(x, y), \tag{10}$$

where L_0 and b are control parameters proportional to the wind force at all altitudes and depending on the height respectively.

The rendering process is then carried out as follows.

Step 1. Normal Vector Calculation - Compute the normal vectors of the sand ripples using the height map $h(x, y)$.

Step 2. Optimization - Perform backface culling and view frustrum culling [17] on each quadrilateral F of the height field mesh. Subsequently, calculate levels of detail (LODs) determined by the distance from the viewpoint to F.

Step 3. Texture Mapping and Bump Mapping - Based on the LODs, assign a texture to each F. Quadrilaterals which are closer to the viewpoint receive bigger texture resolutions, while the farther ones are assigned lower texture resolutions. Also, use the normal vectors calculated earlier to perform bump mapping [18].

This method efficiently simulates the formation dynamics of desert features using height fields and has a primary focus on sand ripple formation alongside dune formation. The result of rendering a dune with sand ripples is shown on Fig. 7.

Fig. 7: A dune with sand ripples rendered running on a Pentium III 750 MHz with a NVIDIA GeForce 256 video card in 1.9 seconds by Onoue and Nishita's physically inspired appromixation model. Adapted from [16].

Building upon the foundational work of Onoue and Nishita [16], Benes and Roa [19] introduced three significant enhancements to the modeling of desert scenes, specifically by integrating the impact of wind obstacles into the simulation. This approach provided a more comprehensive understanding of natural desert formations and their interaction with environmental elements, changing sand ripple formation around objects. The enhanced features introduced by Benes and Roa [19] are described below.

Incorporation of Wind Obstacles - Benes and Roa's model [19] addresses the dynamics of wind interaction with static objects, a factor not previously considered in Onoue and Nishita's approach [16]. For example, if a saltating sand grain would be transported from (x, y) to (x', y') and there is an obstacle at (x', y'), then the sand grain would be deposited on the path from (x, y) to (x', y') at the boundary of the obstacle. By simulating the accumulation of material on the windward side of obstacles and the formation of wind shadows on the leeward side, Benes and Roa's model adds a layer of complexity to the environment. This results in a more realistic depiction of material distribution as influenced by objects within the scene such as cacti or artificial structures.

Material Accumulation and Creep - The concept of creep, as integrated by Benes and Roa, accounts for the gradual movement of sand and its effect on the terrain's topology. The model tests the local gradient (∇) of each element and compares it to the repose angle. If the gradient is greater, then an avalanche occurs in which the height is distributed from that element to its neighboors proportionally to the size of the gradient. This not only enhances the visual fidelity of the simulations, but also aligns with the physical behaviuor of sand in natural settings. The model effectively captures the essence of material buildup against obstacles, contributing to the formation of new landscape features over time.

Wind Intensity and Shadow Simulation - Another novel aspect of Benes and Roa's extension is the simulation of wind intensity decay with distance from obstacles, introducing the notion of wind shadows. This geometric resolution allows for the rapid approximation of complex aerodynamic effects without resorting to computationally intensive fluid dynamics calculations. Benes and Roa present an *ad hoc* technique that produces visually plausible results at interactive frame rates as shown in Fig. 8. Note that this is not a physically accurate approach, which would consider factors such as wind-obstacle interaction, obstacle surface viscosity, wind vortex etc.

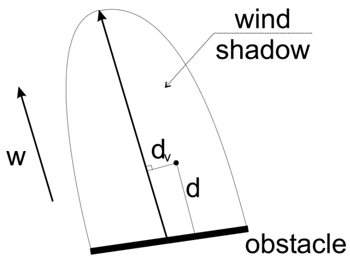

Fig. 8: A sketch depicting the area of a wind shadow behind an obstacle defined by the parabola $s = \frac{w}{2}(1 - d^2)$, where w is a vector representing the wind, d_v is perpendicular to the direction of the wind and d is a control parameter representing the width of the object. Adapted from [19].

Benes and Roa's extension was critical for animating scenes where sand accumulated against obstacles such as buildings or vegetation, a common natural occurrence in sand ripples. It allowed for the creation of more dynamic and realistic simulations of sand movement, showing sand accumulation on the windward side of objects, significantly affecting visualization near obstacles as shown in Fig. 9. Also, the wind shadow effect is evident in Fig. 9, where the sand ripples are noticeably diminished on the leeward side of the cactus. Benes and Roa's enhancements empowers sand ripple enthusiasts to incorporate objects into simulations with greater confidence by beginning to predict how these additions will influence sand ripple formation and enhance the visual realism of wind-terrain interactions.

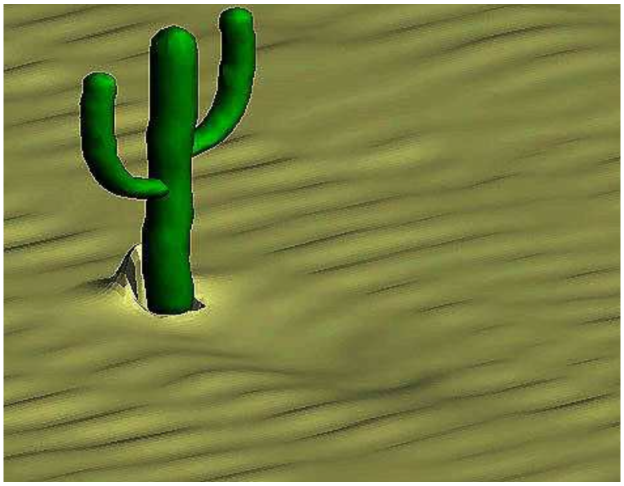

Fig. 9: A *Stenocactus coptonogonus* alongside a wind shadow and sand ripples rendered based on Benes and Roa's enhancements using an IBM PC running on 1.5 GHz and a frame rate between 4-10FPS. Adapted from [19].

To conclude, the works of Onoue and Nishita [16] and Benes and Roa [19] leveraged the principles of the saltation equation and physically-inspired approximation models while paving the way for dynamic simulation of desert environments in computer graphics.

C. Physically Based Modelling

In a pioneering approach to simulate aeolian sand transport, Wang and Hu [20] employed physically-based equations inspired by Bagnold's [1] seminal work on the physics of blown sand. Wang and Hu's work is not only notable for being the first model that extensively integrates Bagnold's principles, but also for introducing a real-time simulation framework. This significantly contributed to the understanding of sand ripple formation. Wang and Hu stand out for their comprehensive treatment of saltating sand grain dynamics, encompassing several key processes, which are outlined below.

Grain Entrainment - Wang and Hu conceptualize a lift-off velocity threshold, beyond which sand grains become airborne. This threshold, grounded in empirical measurements, serves as a cornerstone for initiating the simulation of grain transport.

Transportation in the Air - Once entrained, grains undergo various forces during airborne transit. Wang and Hu focus on gravity and drag forces, discarding less significant forces to streamline the model. This simplification aligns with their goal of real-time simulation without compromising the physics of grain movement.

Collision and Deposition - Upon returning to the ground, the grains' interactions with the sand bed are modeled through collision and damping parameters. These parameters help simulate the complex behavior of sand grains as they settle and contribute to the formation of ripples on the sand surface.

Further enriching their simulation, Wang and Hu [20] represent the sand using a height map, which echoes the methods employed in previous studies [16] [19]. They also integrate a method similar to Musgrave's thermal weathering from their previous work [21] to simulate sand avalanching when too many sand grains accumulate at one place, offering a nuanced depiction of sand behaviour and ripple evolution. Moreover, the wind field model in Wang and Hu's simulation advances the fidelity of environmental factors influencing sand transport. Rather than relying on oversimplified wind models, they implement a logarithmic wind profile founded on boundary-layer theory and incorporate wake flow calculations. This approach not only captures the variability of natural wind patterns, but also enhances the realism of sand movement within their simulation.

Wang and Hu incorporated numerous physically-based parameters in their experiments, enabling them to generate various realistic depictions of sand ripples as shown in Fig. 10. In Fig. 10(b), the wind direction fluctuates, resulting in rope-like sand ripples that differ from the conventional ones in Fig. 10(a), which are formed by a unidirectional wind. Interestingly, an unusual pattern is observed in Fig. 10(d), where a clockwise whirlwind flow creates a unique sand ripple formation, not typically seen in nature. This could be particularly appealing for creating striking visualizations. Their experiments successfully replicated many of Bagnold's findings [1] such as the merging of sand ripples over time and the increase in ripple wavelengths with higher wind speeds.

For rendering the dynamic sand flow and the static sand bed, Wang and Hu [20] adopt a particle system in conjunction with a height field. Their implementation is an extension of the work presented by Kolb *et al.* [22] on state-preserving particle systems, optimized for modern GPUs. A key feature of their rendering technique is the use of double buffering, which allows for the efficient updating of particle positions and velocities without incurring the performance penalties typically associated with read-write cycles to GPU memory [23].

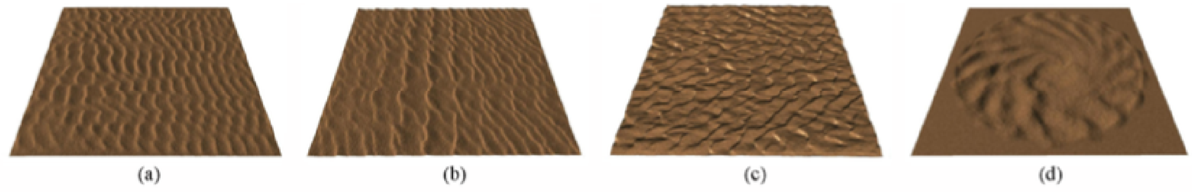

Fig. 10: Different sand ripples rendered by Wang and Hu running on an Intel Core 2 6600 2.4 GHz with a NVIDIA 8800 GTX and a frame rate exceeding 60FPS. (a) Normal sand ripples. (b) Rope-shaped ripples marks. (c) Scale-shaped ripple marks. (d) Whirlwind. Adapted from [20].

To summarize, Wang and Hu's research [20] represents a significant milestone in the real-time simulation of aeolian sand phenomena. Despite the inherent complexity of physically-based equations, their innovative approach leverages the computational prowess of modern GPUs to execute these calculations efficiently. This marks a considerable advancement from earlier physically inspired models, which often simplified physical laws at the expense of realism. The primary objective of Wang and Hu's work was the formation of sand ripples, a goal they achieved with commendable correctness on a small-scale simulation. Their model not only provides visual realism, but also aligns with the physical underpinnings of sand transport and deposition.

D. Towards Large-Scale Real-Time Simulation

Recently, desert models have become even more complex. These include, for instance, the work presented by Paris *et al.* [7]. It employs an interactive simulation with a wind field to depict a realistic desert environment. It allowed for the generation of various types of sand dunes and the simulation of abrasion processes that sculpt bedrock into complex landforms. While the primary focus of Paris *et al.*'s work is on the formation and dynamics of sand dunes, the visualization of sand ripples could be significantly enhanced by incorporating their innovative concepts such as the application of a wind field and stochastic events to simulate the transportation of sand grains.

The model follows an approach similar to the one employed by Onoue and Nishita [16], that is, it also uses a height map, but in addition to a wind field w defined by:

$$w(p,t) = \sigma(p,t)\omega \circ a(t) + u(p,t), \tag{11}$$

where $w(p,t)$ denotes the wind vector at point p and time t, $\sigma(p,t)$ symbolizes the wind shadow field, ω represents surface warping, $a(t)$ represents the high-altitude wind field and $u(p,t)$ is the user controlled perturbation field, as shown in Fig. 11.

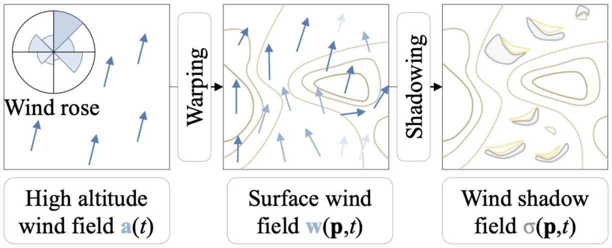

Fig. 11: Overview of the wind field computation. The wind vector w is formed by warping the high altitude wind field a to consider the variation in wind over different terrain, and then shadowing it with the wind shadow field σ which represents the reduction in wind speed behind obstacles. Adapted from [7].

The high-altitude wind field a is characterized as a time-varying vector field of uniform direction and speed, depicted in a wind rose diagram presented in Fig. 11. The wind rose encapsulates the directional distribution and velocity of wind at higher altitudes, providing a foundational input for the model. This high-altitude wind field is then warped according to the relief of the terrain at different scales, thus accommodating the complex topology of natural desert landscapes. The warping effect ω in the simulation accounts for the variation in wind direction and speed over different terrain features. Paris *et al.* [7] recognized that in natural environments, wind does not flow uniformly, but it is influenced by the topography it encounters. To simulate this, they used a multi-scale warping function that adjusts the wind field locally, providing the ability to recreate complex wind patterns observed around natural features such as dunes, vegetation and other obstacles. This approach enables the wind to accelerate over peaks and slow down in valleys, imitating the Venturi effect[3] observed in real-world scenarios [24]. Wind shadowing σ is another critical aspect explored in their work, representing the reduction in wind speed behind obstacles. This phenomenon is crucial for understanding sand deposition patterns, as areas in the lee of obstacles tend to accumulate sand, leading to the formation of leeward dunes. By calculating the wind shadowing based on a 15-degree accessibility angle, the model ensures that wind-borne sand is realistically deposited downwind of obstacles, shaping the dunes accordingly. To complete the wind field, a controlled perturbation field u is introduced to allow users to manipulate the wind field and simulate specific dune formations.

The model proposed by Paris *et al.* [7] also includes the concept of avalanching, extending the use of Bagnold's work [1] on granular movement and further enriching the simulation of sand behaviour. The system could calculate wind fields over the terrain, simulate sand transport by the wind and represent the interaction between sand and static elements. They incorporated stochastic events to simulate saltation and reptation processes as an additional contribution. Saltation was modeled as a series of probabilistic events that dictate whether a grain bounces or is deposited once it hits the ground based on wind shadowing, the amount of sand grains and the vegetation density. As for reptation, the model accounts for the likelihood of grains moving laterally solely depending on the vegetation density. Incorporating these stochastic elements into the simulation of saltation and reptation allows for the simulation of the random nature of sand grain trajectories seen in real environments.

The inclusion of a high-altitude, time-varying wind field in the model proposed by Paris *et al.* [7] represents a major advancement in the visualization of sand ripples, offering more detail and realism than previous methods [16] [19] [20]. Applying this approach to sand dunes to visualize sand ripples, as shown on Fig. 12, suggests that smaller-scale models could also benefit from integrating a wind field to visualize sand ripples at varying altitudes and in interaction with diverse objects. Furthermore, the incorporation of stochastic elements into the simulation of sand grain dynamics can be seen as an original contribution since it has not been previously considered in this area. Applying this realistic concept to smaller-scale visualizations would be an interesting and valuable advancement.

Overall, their approach is much more general and contributes to a wide range of desert features instead of solely focusing on sand ripple formation. In fact, sand ripples are considered as smaller details since they are generated procedurally as a post processing step with smaller obstacles. The final sand elevation is defined as:

$$S' = S + SR + SB, \tag{12}$$

where SR and SB denote the sand ripples and sand bumps caused by small obstacles respectively. Sand ripple sizes are determined linearly to the wind speed, and parallel asymmetric ripples are oriented orthogonally to the wind direction.

Despite its sophistication, the desertscape simulation carried out by Paris *et al.* [7] had limitations such as resolution constraints and the need for computational efficiency to maintain real-time interaction. That

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[3] The Venturi effect refers to the phenomenon where fluid pressure decreases as the fluid flows through a constricted section of a pipe or a channel. In the context of wind flow, as wind enters a narrow passage formed by natural features, such as mountains or valleys, its speed increases and the pressure decreases [7].

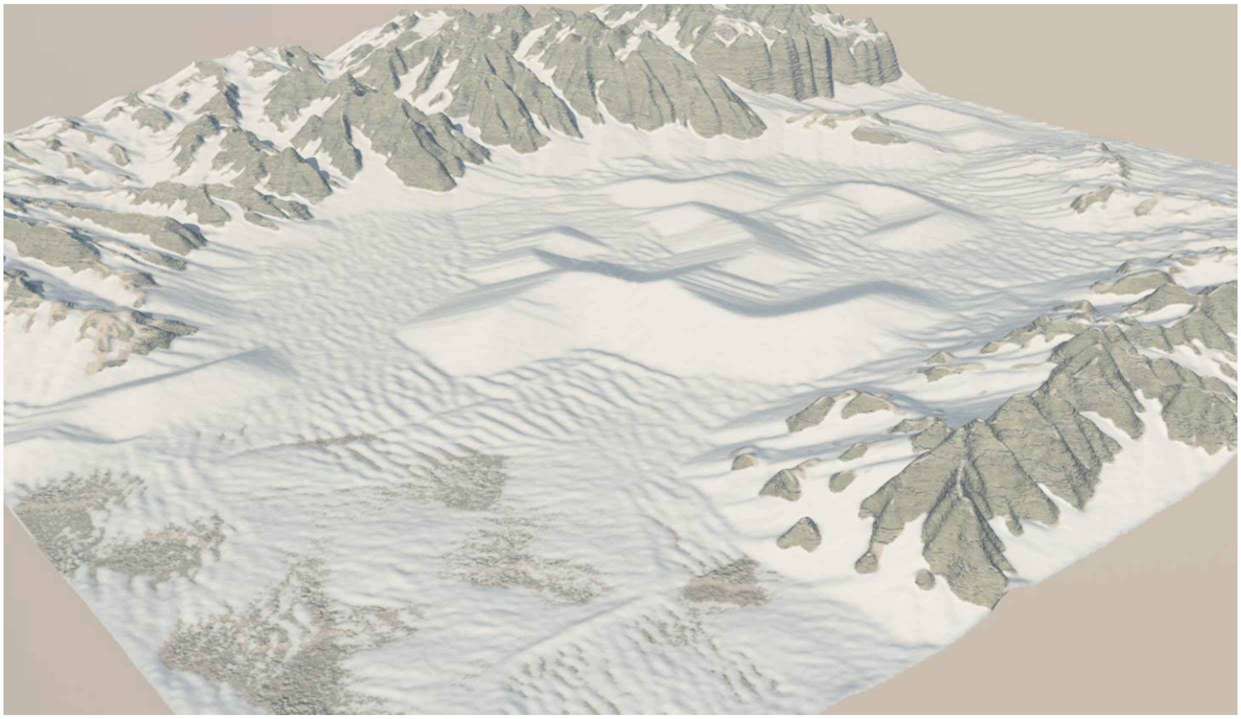

Fig. 12: A desert landscape with sand ripples modeled using Paris *et al.*'s approach running on an Intel Core i7 4GHz with a NVIDIA 970 GTX. Adapted from [7].

is exactly what Taylor and Keyser addressed [10], bridging the gap between event-driven and real-time graphical simulation for the model proposed by Paris *et al.* [7]. By advancing the model on the GPU and extending it to include obstacle interactions, Taylor and Keyser broadened the horizon for believable, real-time visualization of aeolian processes as shown in Fig. 13. In Fig. 13(a), the transverse dunes form under consistent wind directions and abundant sand. Unidirectional wind will result in crescent shaped barchan dunes in Fig. 13(b), whereas winds blowing from multiple directions will produce the star dunes seen in Fig. 13(c).

Fig. 13: Different types of sand dunes generated by Taylor and Keyser's GPU implementation [10] (running on an i7-8700 CPU with a NVIDIA TITAN V) based on the model proposed by Paris *et al.* [7]. Left: transverse dunes. Middle: Barchan dunes. Right: star dunes. Adapted from [10].

The simulations conducted by Taylor and Keyser [10] yield insights into the overarching morphology of large-scale desert landscapes, encompassing a vast array of dune types and wind-driven processes. While sand ripples are an inherent component of these landscapes, they are not the focal point of their contribution. Demonstrating that large-scale desert landscape simulations can be conducted in real time implies that smaller-scale simulations of sand ripples, even with complex factors like wind fields and stochastic events, are also feasible in real time. Smaller-scale simulations would require fewer resources, indicating that there should be no significant limitations in visualizing sand ripples using these techniques.

IV. DISCUSSION

A. *Balancing Realism and Computational Efficiency*

In selecting an existing algorithm for the depiction of sand ripples, the decision must be carefully aligned with the project's specific requirements. If the primary objective is to achieve the utmost realism, Wang and Hu's physically-based model [20] stands as a suitable choice. Its comprehensive treatment of the physics of sand movement provides an intricate level of detail that is paramount for simulations that aspire to closely mimic real-world phenomena.

While the large-scale environmental models [7] [10] are undeniably impressive, their focus is not exclusively on the intricacies of sand ripple formation. Nevertheless, the methodologies they introduce can be invaluable when adapted to the finer scale of sand ripples. An innovative approach could involve integrating the complex wind fields from Paris *et al.*'s model [7] into Wang and Hu's physically-based framework [20], potentially creating a more dynamic and authentic simulation. Additionally, introducing elements of stochastic behaviour into the simulation could reflect the inherent randomness present in natural sand ripple formation, thus enhancing the realism of Wang and Hu's model.

For projects where expediency and simplicity take precedence, or where computational resources may be more constrained, adapting simpler models [16] [19] to leverage modern GPU capabilities could yield a significant improvement. This would allow for real-time simulations that maintain a balance between visual believability and computational efficiency, making use of physically inspired approximations to model the essential dynamics of sand ripples. Such adaptations would make these earlier models more accessible for a wider range of applications, including interactive media and educational tools, where user engagement and real-time feedback are crucial.

B. *Research Perspectives*

To effectively render high-resolution sand ripples in real time, we must acknowledge the limitations in current techniques. The real-time generated sand ripples, as per Wang and Hu's method [20], offer low resolutions (512 X 512) and are oversimplified, lacking considerations for wind field and stochastic events. Meanwhile, larger scale models by researchers like Paris *et al.* [7] do not adequately focus on sand ripple details, resulting in less effective visualizations.

Addressing these issues opens up exciting research opportunities. One avenue is to leverage modern GPUs and parallel computing to enhance Wang and Hu's physically based model [20] alongside the implementation of a wind field and stochastic events for sand transport. This could enable higher resolution and more detailed renderings. However, if this approach proves too computationally intensive as Wang and Hu have already simplified their model for performance purposes [20], alternative strategies can be considered. First, one could train a deep learning model using historical data on sand particles. This strategy could reduce the computational burden by enabling the model to predict and render sand ripple patterns more efficiently. Second, one could develop a model that incorporates simplified equations such as ones based on wind field dynamics and stochastic events affecting sand grain movement. This would enable a more cost-effective representation of the natural variability and complexity of sand ripple formation. The ultimate goal in this research should be to find a sweet spot between computational complexity and realism. This balance is crucial for achieving the most effective and realistic rendering of sand ripples, which can be pivotal for the success of a broad range of applications, from environmental monitoring to educational simulations.

V. CONCLUDING REMARKS

In summary, our study has articulated the significant strides made in the computational visualization of sand ripples within the realm of computer graphics. Beginning with procedural techniques and evolving through physically inspired models to sophisticated physically based and large-scale real-time simulations, the trajectory of development is clear and impressive. Choosing an algorithm for representing sand ripples hinges on the project's scope, *i.e.*, whether it demands the highest level of realism or the efficiency and simplicity of an *ad hoc* method.

Thus, through the integration of pioneering scientific principles into advanced computer simulations, the field has achieved remarkable strides in the realistic and real-time depiction of sand dynamics, opening new vistas for both scientific inquiry and visual storytelling in digital environments. Reflecting the evolution of research focus, simulations have predominantly concentrated on aeolian sand ripples found in desert environments. This aspect, in turn, offers a rich opportunity for future computer graphics research on subaqueous ripples.

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