Simulation of the ball lightning phenomenon

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Abstract

Rendering images of outdoor scenes capturing the magnificence of the natural world is a challenging task for computer graphics. One natural phenomenon that has received no attention in the field of computer graphics and scarce attention in physics is ball lightning. A computer simulation would greatly improve animated sequences of thunderstorms where this rare feature most often occurs. A physical model of ball lightning may also aid physicists in discovering its true nature. This paper describes the first computer graphics simulation of ball lightning. It is based on observations and physical theories which have attempted to describe its properties. None of the theories presented so far can explain all the manifestations of ball lightning reported in the literature. Consequently, it was not possible to develop a purely physical simulation; however, an attempt was made to include as many of the reported manifestations as possible. Common computer graphics techniques that are efficient and easy to implement are used to approximate the shape and visual appearance, as well as the deformations of a ball lightning as it passes through a small opening. An emphasis is placed on clearly defining a set of parameters that affect the visual qualities of the animation. In this way, the final output can be adjusted to suit the variety of observations that have been documented. Since ball lightning research is new to the field of computer graphics, an introduction and survey of the current state of ball lightning research is included.

Keywords: Computer graphics; Object modeling; Physically based modeling

1. Introduction

Computer graphics is about simulating the world graphically. The world can be partitioned into artificial or man-made objects and natural objects. The properties of man-made objects such as surface characteristics are often known from their manufacture and hence, the graphical simulation of such objects is relatively well developed. Graphical simulation of natural objects is more difficult since their characteristics are often not known and nor are their properties always fully understood.

The reasons for the lesser understanding of natural phenomena is that they might not be reproducible in a controlled lab situation where their properties could be elicited. Field measurements are difficult to execute and frequently compounded by the often very short lifetimes of many of the objects to be investigated. Still, brief highly destructive phenomena such as lightning have received a lot of attention due to their immediate threat to sensitive electronics and even life [1]. Common phenomena having a high visual impact such as rainbows have been reproduced since they have also been studied extensively for a long period of time [2]. Less common phenomena have been largely ignored by the scientific community. This is partially due to their relative rarity, but also due to neglect. Such phenomena are important as they may help to predict cataclysmic events such as earthquakes [3].

Other than the obvious applications in the entertainment industry, there is also merit in graphically simulating natural phenomena to test physical theory. Plotted solutions to dynamical systems yield a static graph that does not give a strong temporal impression of the results. By realistically animating the same solution, a strong impression is made on the observer. Furthermore, untrained eyewitnesses of a rare event can easily affirm or deny the realism of an animation; whereas only a trained professional can understand the plotted output of a complex mathematical integration.

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2. What is ball lightning?

Ball lightning is a rare natural phenomenon. Reported properties vary, but modal characteristics have been determined from several surveys [4]. Typically, ball lightning is associated with thunderstorms of average violence. It is usually spherical in form, but other shapes have also been observed. It usually moves horizontally. Sometimes ball lightning movement is reported to be independent of the environment, even against the wind. Ball lightning has entered enclosed buildings through small openings, and has been documented to pass through glass panes [5]. Most observations report that the diameter is between five and fifty centimeters, and that it is bright enough to be seen in daylight. Larger lightning balls have been observed to deform so that they may pass through small openings and walls [6]. It has even been seen abroad an aircraft [7]. Sometimes it decays silently; other times it explodes violently. There is evidence to suggest that it can contain a considerable amount of energy, since it has reportedly damaged objects and killed people. On the other hand, some people have been touched by ball lightning without injury. Most observations last for less than ten seconds. Ball lightning is often “flame coloured”, but can also be white, red, blue, or green [8]. Usually, ball lightning is described as being one solid colour. Occasionally, an internal bead-like structure has been observed [9,5].

Systematic studies of this elusive phenomenon have been unsuccessful. Not a single photograph has been produced in this manner, though a few photographs exist from chance observations (see Fig. 1 for example). As a consequence, ball lightning theory is largely based on the observations made by untrained observers; though several reports have come directly from the scientific community. Despite the rarity of the phenomenon, some 10,000 reports have been collected over the last two centuries [8].

Many theories have been proposed to explain the ball lightning phenomenon. In general, there are two categories to consider. Some theories assert that ball lightning has a completely self-contained energy source. For example, ball lightning events may be explained with a model based on combustion of organic material [11] or on unusual chemical reactions [12]. There are also many more dramatic models. For example, Altschuler et al. [13] theorize that ball lightning is a nuclear phenomenon. The other class of ball lightning theories uses an external energy source to power the phenomenon [14]. A more skeptical theory presents ball lightning as an optical illusion [15] along the lines of the perceived, but not actual, larger size of the sun disc as it passes below the horizon [16].

3. Plasma

Plasma is often called the fourth state of matter. It is the highest-energy form of matter that retains its chemical properties [17]. When enough energy is introduced to a gas so that electrons become free from the nuclei, the electromagnetic properties of the material are altered. Recombination of the electrons with the nuclei will give off a photon. These properties of plasma make it ideal for many ball lightning theories.

The majority of the universe is in the plasma state; but near the Earth’s surface, plasma only exists naturally within lightning channels. The Northern and Southern lights in the ionosphere are also examples of natural phenomena that are due to matter in the plasma state.

A large number of ball lightning models suggest that plasma is the source of the luminescence of the phenomenon (see [4]). Plasma models attempt to explain how the plasma can be contained, resist convection, and decay slowly. For example, convective forces may be balanced by the electric fields of the a thunderstorm [4].

This work will assume that ball lightning is a contained plasma, and that the internal structure may be visible at close proximity.

4. Plasma-based phenomena

There are at least two other natural phenomena in the Earth environment that are known to exist in the plasma state: lightning and aurorae. Since this work assumes that ball lightning is a plasmoid, the animation and rendering techniques used for these plasma-based phenomena need to be reviewed.

4.1. Lightning

Most lightning strokes that connect a cloud to the ground are formed when a negatively charged leader descends from a cloud (see Fig. 2). This leader will induce a positive charge below the Earth’s surface, which behaves like an electrical mirror. As the induced positive charge grows on the Earth, streamers of charge form in the air.
When the descending negative leader makes contact with one of these streamers, the circuit is completed and the negative charge in the cloud is neutralized. There are often several strokes of lightning for every flash that is observed [1]. Since this is the most dramatic form of lightning that is commonly observed, it is the type of lightning that has been concentrated on in computer graphics. Infrequently, lightning may also have a positively charged leader with a negatively charged streamer.

Computer graphics models tend to focus on the shape of the lightning channel. Generally, a channel is divided into segments. Segments are chosen sequentially until the channel terminates at the ground. Lightning flashes are extremely bright, and very short-lived. Furthermore, plasma recombination of a stroke takes less time than is allocated for an individual frame of an animation. (A stroke lasts approximately 1 ms versus 40 ms allocated for each frame of a 24 frames per second animation.) Consequently, there has been no attempt to directly simulate the nature of the plasma for the purpose of rendering lightning.

Reed and Wyvill [18] presented the first computer graphics model of lightning. Their model presents a stochastic method for generating the channel of a plausible stroke of lightning. They also present an exponentially decaying fog function that can be used to render a stroke of lightning. Another stochastic model has been proposed by Kruszewski [19]. This model provides an improved stochastic algorithm that gives more control over the resulting stroke. It also proposed a mechanism to add more detail to the lightning channel, which Kruszewski calls “electrification”. In the same year, Glassner [20] provided his own stochastic model which is based on measured data from lightning strokes. A more physical model has been presented by Sosorbaram et al. [21]. Their model is a general simulation of electric discharge, and can be used to simulate various types of lightnings by placing charges in the appropriate locations of a voxelized volume. Leaders are propagated from the negative charge location until contact is made with a positive charge center.

4.2. Aurorae

The aurora borealis and the aurora australis (more commonly known as the Northern and Southern lights) are also plasma phenomena. The source of these two phenomena is the sun. High-energy particles are ejected from giant storms on the sun. These are known as coronal mass ejections. The hurtling plasma is called the solar wind. When the solar wind encounters the Earth’s magnetic field, the field traps the plasma particles. At the same time, the field is deformed away from the sun forming the magnetosphere, a shell of energized particles at the outer reaches of the Earth’s atmosphere. Events known as substorms [22] then eject these particles into the upper atmosphere where
they energize atmospheric constituents creating free electrons and ionized nuclei. Recombination of the new free electrons with atomic nuclei produces the light at different frequencies depending upon composition of the atmosphere.

A very detailed physical simulation of this process has been performed by Baranoski et al. [23,24]. In their work, a serious attempt has been made at modeling the detailed interactions of the plasma particles. A sinusoidal function is used to represent the shape of the solar wind front interacting with the atmosphere. Parameters are used to control the period and to add knots to the function, which are commonly observed and physically justified. For efficiency reasons, the simulation is not to be performed at the atomic level. To make the problem more tractable, beams of particles are considered in parallel.

The particles of the beam energize atoms of the atmosphere. After a number of interactions, the ionized atoms might recombine with free electrons and in the process eject a photon. The effect of this is that each beam acts as a light source as it is projected onto the view plane of the observer.

5. Ball lightning model

This section presents a method for animating a ball lightning that moves through the air and then passes through a small opening. The model presented here is divided into two distinct parts: non-deformable motion through the air, and deformation through the opening. Gaidukov [6] presents the fluid dynamical justification of how this can occur. His physical description makes several assumptions. Consequently, this work must also make the same assumptions. They are:

- the path of a ball lightning is not guided by electromagnetic fields,
- the ball lightning has approximately the same density as the surrounding air,
- an interface boundary layer separates the internal structure of a ball lightning from the environment, and
- the radius of the ball lightning remains constant as it moves through the air.

The first assumption does not affect the rendering since electromagnetic fields are not visible and since the assumption is that these fields do not affect the motion of the ball lightning. The physical density of the phenomena is not used in the rendering calculations and as such the second assumption also does not affect output images; though, if this assumption is violated, the path of the ball lightning would certainly be altered. The third assumption implies cohesion, and disregards any internal features. The final assumption prevents simulations where the ball lightning changes size. No mechanism is provided that can be used to generate how the ball lightning phenomenon actually looks.

5.1. Non-deformable motion

Gaidukov presents a dynamical system that accurately describes the motion of a ball lightning that is distant from a small opening, i.e. one ball-lightning radius away. At such a distance, a point source can be used to draw the ball lightning toward the hole, instead of a jet. Also at this distance, a ball lightning can be assumed to be non-deformable. These two added assumptions simplify the dynamical system.

Gaidukov’s equations of motion for a non-deformable ball lightning far away from a point source are:

\[ \ddot{r} - r(\dot{\theta}^2 + \dot{\phi}^2 \sin^2 \theta) = -\frac{2\gamma}{a^5} \left[ \frac{\alpha^5}{r^7} + 2 \frac{\alpha^7}{r^9} \right], \]

\[ 1 \frac{d}{dr} \left( r^2 \dot{\theta} \right) - r \dot{\phi}^2 \sin \theta \cos \theta = -\frac{r \theta}{\rho^2}, \]

\[ 1 \frac{d}{r \sin \theta} \left( r^2 \dot{\phi} \sin^2 \theta \right) = -\frac{r \dot{\phi} \sin \theta}{\rho^2}, \]

where \( r, \theta, \phi \) is the center of the ball lightning in spherical coordinates, \( a \) is the ball lightning radius, and \( \gamma \) is the point-source intensity. The position varies with time, but \( a \) and \( \gamma \) are constants (recall that \( \dot{r} \) means \( dr/dt \) and that \( \ddot{r} \) means \( d^2r/dt^2 \)).

The path of a ball lightning can be obtained by solving this system of ordinary differential equations as an initial-value problem. Implementing a numerical solution for this purpose is easy and efficient, since extremely high accuracy is not required for computer graphics [25]. A fixed-step fourth-order Runge-Kutta method was used to solve the system. Six scalar values must be provided in the first step of the solution for this initial-value problem—three for the initial position and three for the initial velocity. The acceleration is computed as the differential by the Runge-Kutta method.

Spherical coordinates are convenient for physicists, but they are not practical for computer graphics. Cartesian coordinates are much easier to specify. The conversion factor for a 3D position from Cartesian to spherical coordinates is well known:

\[ \begin{pmatrix} r \\ \theta \\ \phi \end{pmatrix} = \begin{pmatrix} \sqrt{x^2 + y^2 + z^2} \\ \tan^{-1} \left( \frac{y}{x} \right) \\ \cos^{-1} \left( \frac{z}{\sqrt{x^2 + y^2 + z^2}} \right) \end{pmatrix}. \quad (1) \]

To convert the initial velocity from Cartesian to spherical velocities, each row of the above system must be differentiated. Applying the chain rule to each of the rows
in Eq. (1) yields the following conversion formula:

\[
\begin{pmatrix}
\frac{x}{r} \\
\frac{y}{r} \\
\frac{z}{r}
\end{pmatrix} = \begin{pmatrix}
\frac{1}{r} (x \hat{x} + y \hat{y} + z \hat{z}) \\
\frac{1}{1 + (y/x)^2} \left(\frac{-y}{x^2} \hat{x} + \frac{1}{x} \hat{y}\right) \\
\frac{1}{r^2 \sqrt{1 - z/r}} (xz \hat{x} + yz \hat{y} - (x^2 + y^2) \hat{z})
\end{pmatrix}, \tag{2}
\]

Note that \( r = \sqrt{x^2 + y^2 + z^2} \) from Eq. (1).

A sample plot is provided in Fig. 3. There the ball lightning starts at position \((4, 3, 1)\) and has an initial velocity of \((-2, 0, 3)\). The point source is located at the origin.

5.2. Deformation through an opening

To simulate the deformation of a ball lightning through a small opening, the area surrounding the opening is discretized into voxels. The dimensions of the voxel volume are determined from the radius of the ball lightning and the desired path length of the approach. Since the size of the voxel volume is fixed, simple arithmetic can be used to convert from real-world coordinates to a voxel index and vice versa. Shared pointers to voxels are stored at each index of the volume so that only unique voxels need to be allocated. This reduces the total memory requirements.

Fig. 4 demonstrates the volume that should contain the ball lightning as it passes through the opening. The discretization of this volume is divided into four segments. The voxels in each segment are initialized with a velocity vector as appropriate for that segment. To actually simulate the deformation of a ball lightning through the opening, a particle system is advected through the voxel volume. Each particle’s position is computed sequentially for each frame of the animation.

The purpose of the first segment is to move the ball lightning forward toward the opening. Each voxel is initialized to \((0, 0, -s)\), where \(s\) is the desired speed of the ball lightning per frame.

To acquire the desired shape of deformation, a monotonically increasing function is convolved around the \(z\)-axis. (Note that the positive \(z\)-axis points to the left in the Fig. 4.) A suitable function must be continuous and have a slope of zero at \(z = 0\) and \(z = 1\). The domain and range of the function should both be \((0, 1)\) in order to be suitable for scaling. The function \(b\) should have the property that \(b(0) = 1\) and \(b(1) = 0\). Field functions (sometimes called “blob” functions) are well suited for this purpose, when reflected in the \(y\)-axis. The field function provided by Baranoski and Rokne [26] was chosen since it is controllable and efficient to evaluate. Fig. 5 demonstrates how the field function is convolved around the \(z\)-axis.

Notice in this diagram that the length of the hypotenuse can be computed using the Pythagorean theorem in \((x, y)\),
or by evaluating the reflected field function at z. Thus, if the difference of these two is set to a constant \( k \), an implicit surface is defined,

\[
k = \sqrt{x^2 + y^2} - b(z).
\]

(3)

By varying \( k \), a set of concentric surfaces are implicitly defined. These surfaces will be used to initialize the second segment of the voxel volume. Consequently, we define the following function:

\[
F(x, y, z) = \sqrt{x^2 + y^2} - f(z).
\]

(4)

In Eq. (4), \( f(z) \) represents the blob function that has been reflected in the \( z \)-axis, and scaled to suit the diameter of the ball lightning. The gradient of \( F \) can be used to compute the normal of the concentric surface for any given point \( p \) in space. This normal is used to define a tangent plane at \( p \). A vector that points to the origin from \( p \) is then projected onto this plane to determine the direction vector for the voxel at \( p \) for the third segment voxels in Fig. 4. The complete algorithm is presented below:

**Algorithm 1.** Compute direction vectors for voxels in segment two.

**Step 1.** For each voxel, compute its location, \( p \).
**Step 2.** Compute the normal vector \( n = \nabla F(p) \).
**Step 3.** Create a vector \( t = O - p \), where \( O \) is the origin.
**Step 4.** Compute the direction vector \( d = \text{proj}_p t \), where \( \pi \) is the plane defined by the normal \( n \) and the point \( p \).
**Step 5.** Let \( d = s(d/||d||) \), where \( s \) is the particle speed.

The third segment needs to advect the particle system forward through the opening. The speed of a fluid in a pipe increases linearly with distance from the surface of the pipe [27]. Without knowing the viscosity of the material composition of ball lightning, some assumptions have to be made. This work presumes that ball lightning has a very low viscosity. As such, the material in the center of the opening travels only slightly faster than the material at the boundary. So, for the third segment of Fig. 4, the advection vector is \((0, 0, -\mu s)\), where \( \mu \) is a factor that is scaled linearly between the slowest particle speed at the boundary and the fastest speed in the center along the \( z \)-axis.

A mechanism similar to segment two can be employed for the fourth segment in Fig. 4. This would be more realistic, but this work suggests that a gross simplification be made. The ball lightning is simply allowed to fill a spherical bounding volume via stochastic motion of the units of plasma. The observer of the ball lightning never changes location since the lifetime of the ball lightning is very short, which means that it will have almost no effect on the resulting animation.

To reduce computation time, a bounding sphere is created with the same radius as the ball lightning. Once particles enter segment four, they are permitted to move randomly within the bounding sphere.

**6. Rendering**

Simulating the detailed interactions between ionized particles and free electrons is an intractable problem. Consequently, several independent units of plasma are rendered as a compromise. This prevents a detailed electrochemical simulation from being performed. Such a simulation would not be possible even if the computational power was available. This is because the detailed electrochemical nature of ball lightning is not understood.

A particle system [28] is used to represent the individual units of plasma. Close encounters with some ball lightning events suggest that ball lightning has an internal structure that resembles “dots” [5]. A particle system can be used to produce an effect which resembles this internal structure.

During the non-deformable motion phase of the simulation, plasma units are assigned a direction vector stochastically. They continue to move in their direction until they reach the containing bounding sphere that is defined by the ball lightning’s position and radius. When a particle’s path intersects the bounding sphere, a new direction vector for the particle is chosen as a random offset from the inverse normal of the sphere at the point of contact. The speed of each particle is set to a constant. This constant affects the internal structure of the ball lightning. If the speed is zero, then there is no internal motion and the ball lightning has a static look. If particle density is low, then a low particle speed is distinguishable from one rendered with a high particle speed. With a large number of particles, it is more difficult to discern between a low and a high particle speed.

When the ball lightning is deformed through an opening, each particle in the system is advected in a direction determined by the voxel that contains it. A small amount of randomization is introduced to the particles’ position to represent the particles’ location. This prevents a detailed simulation from being performed. The \( z \)-buffer is stored for the second pass, which renders the ball lightning plasma units at their current position. Each unit is forward-mapped onto the image plane using standard techniques [30]. The \( z \)-buffer is used to determine which particles are occluded, and hence should not be rendered.

Ball lightning can manifest in a variety of colors. Some eye-witness accounts portray a layered structure—sometimes with different colors for the layers [9]. Such an effect can be produced by specifying major and minor radii, and different colors for each. The major radius contains the entire mass of the ball lightning; and the minor radius is slightly smaller so that it contains the majority of the mass, but not all of it. This system can be used to simulate an outer shell for the ball lightning. Particles in this shell can be rendered in a different colour than the particles in the core.
7. Results

The goal of this project was to produce an animated foul weather sequence where a ball lightning is observed. Physical realism is valued over efficiency, and image quality is preferred over real-time rendering. Furthermore, the model must be parameterized so that adjustments can be made to match observations. Fig. 8 presents a few sample frames from one such animation.

7.1. Parameters

There are several parameters relating to the particle system configuration that effect the resulting images. Among these are colour, opacity, and number of particles. The best results were obtained when the number of particles approached 100,000, the colour was fairly dim, and their opacity was 0.15 on a normalized scale.

The speed of the particles affects the animation of the internal structure of the ball lightning. A chaotic look was obtained when the particles were set to traverse a distance equal to the ball-lightning radius in one second.

The initial conditions to the numerical solution determine the starting location and velocities of the ball lightning. The point-source intensity determines how quickly the ball lightning will be drawn toward the opening. Choosing initial conditions and a point-source intensity are entirely dependent upon the desired dimensions of the scene.

The field function used to shape the deformation of the ball lightning is parameterized. The field parameter $d$ must be in the range of $(-1, \infty)$. Fig. 6 demonstrates the results of varying this field parameter.

The lengths of the major and minor radii used to define the ball lightning colours can be used to create a variety of results. For demonstrative purposes, a glow width parameter $g = M - m$ is defined, where $M$ is the major radius and $m$ is the minor radius. Fig. 7 demonstrates the results that are obtained when the glow width parameter is varied. The blue envelope surrounding the ball lightning is sometimes observed by eyewitnesses.

7.2. Simulation efficiency

The numerical solution to Gaidukov’s equations can be solved efficiently since the accuracy required for computer graphics is limited. If a high degree of accuracy is desired for a real-time rendering, the computation can be performed as a preprocessing step. The example solutions presented in this work required only a few seconds to complete.

Simulation of the deformation is memory intensive. A high-voxel-volume density is required for the simulation. For example, the animation sequence in Fig. 8 used over one million voxels with a side length of 0.25 cm to render a ball lightning with a 10 cm radius. Iterating over the particles accesses the voxels (and thus pages of memory) randomly, so caching algorithms will fail for each iteration step. Increasing the density of the voxel volume increases the memory requirements cubically. The memory requirements increase linearly with the number of particles.

Increasing the number of particles increases the time to compute the deformation of the ball lightning linearly. Increasing the voxel volume size does not affect simulation time appreciably, though the time required to allocate memory and initialize the voxels with an advection vector increases to the cube.

7.3. Rendering efficiency

Rendering particle systems with a forward-mapping projection is known to be efficient. Time trials determined...
that 200,000 particles were easily rendered in less than a second on a 1.4 GHz processor. In contrast, phase 1 of the rendering process, which outputs the background scene using traditional ray-tracing techniques, took well over an hour on the same machine. (An area light composed of a 20 × 20 sample grid was used to create soft shadows. Note that dedicated graphics hardware could be used to generate the background image with some loss of realism.) To decrease rendering time, each required background image was rendered only once. Then each frame of the ball lightning simulation was rendered. The final image is a composite of the background scene with the ball lightning.

If preprocessing is acceptable, then the entire animation could be rendered in real-time using specialized graphics hardware on a system with a sufficient amount of main memory. The voxel volume initialization would be performed offline and loaded on startup. Memory allocation for the voxel volume would need to be done on startup, and a second thread can free unused voxels as the animation continues beyond them.

8. Conclusions

This paper presents the first attempt at a computer graphics model for simulating ball lightning. The motion of a ball lightning in the force field of a current is computed by numerically solving a set of ordinary differential equations that have been provided in the physics literature. Simple computer graphics techniques are used to approximate the deformation of the ball lightning through a hole in the wall. The shape of the deformation can be controlled by modifying a single parameter; thus, providing a mechanism that can be used to match the approximation with observed ball lightning events.

Simulating and rendering of the ball lightning is efficient, robust, and easy to implement.

This paper contributes to ball lightning research by attempting to validate a dynamical system of equations provided in the literature. Numerically solving the system yielded a natural-looking, curvilinear path that described the ball lightning motion. However, in the literature it has been noted that ball lightning sometimes follows a path with sharp changes in direction. The dynamical simulation investigated in this work is not able to reproduce such sharp changes in direction. A more complex system is required to produce such effects.

The model described in this work has obvious contributions to the computer graphics industry. An animation of a ball lightning can be used in flight simulators since ball lightning events have been reported by pilots. Furthermore, a ball lightning could be used to add drama to an animated movie feature. Other applications for this simulation can be found by physicists studying the nature of ball lightning. It would be a useful tool to allow eyewitnesses of ball lightning events use the visual parameters described in this work to generate a picture of what they saw. Written and verbal accounts from laypeople who have seen the phenomenon would be much more helpful to researchers if a picture could be used to support their story.

Ball lightning is a very elusive phenomenon. The model proposed in this work will hopefully help to shed some light on its nature.

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References


